Mini-Lab 2: Bias and Precision Errors, and Accuracy

TITLE

Bias and Precision Errors, and Accuracy

OBJECTIVES

- Become familiar with concepts of bias and precision errors.

- Learn use of Excel spreadsheet and functions for statistical analysis.

- Learn how to apply calibration measurements: perform dimensional measurements on a local standard (a gage block) to determine the overall instrumental error and the accuracy of your measurements.

- Measure the dimension of an "unknown" object: perform dimensional measurements on an object, determine the "unknown" dimension, and give the total error and accuracy of the measurement.

Before beginning the lab, you should familiarize yourself with how bias and precision errors are determined by reading **Appendix A** at the end of this lab handout. Proceed to the calibration measurements.

A.　Calibration Measurements

Select one gage block between 0.1 to 1 inch from the gage block set. The gage block will be the **standard** of your measurements.

Using a digital calipers and a mechanical micrometer, measure the thickness of the gage block. Take at least forty readings using each device, recording them in your logbook. The measuring point is to be made at the "center" of the gage block.

Perform the following tasks and report the results you obtain in your logbook:

(a) Open Excel and create a spreadsheet to analyze the data.

(b) Compute the sample mean, variance, and standard deviation for each measurement device;

(c) Determine the precision error for both devices, based on a 95% confidence level. *Hint: this comes from the statistical analysis of the repeatability of your measurements.*

(d) Using the gage block thickness as the standard value, determine the bias errors for both measurement devices. *Hint: these come from the accuracy of your standard, the bias error of the mean value of your measurements relative to the standard value, and the resolution of your measurement device.*

(e) Determine the total measurement error (overall instrumental error) from the calibration measurements using the root sum of the squares of the individual errors.

(f) From the total instrumental error, determine the accuracy of your measurements for each device relative to the standard gage block thickness (the "true value").

Perform all necessary calculations in your spreadsheet, and summarize your calculations and results in your logbook.

TECHNICAL INFORMATION OF GAGE BLOCKS

The ambient temperature must be maintained constant at 20 ˚C for accurate measurements and the gage blocks should have reached thermal equilibrium with the ambient temperature before taking measurements. The gage blocks (Mitutoyo 516 Series) in this laboratory has a certified Grade 2 accuracy [1]. Handle the gage block carefully and never damage its surface, particularly the measuring face. Do not drop a gage block even onto a soft surface and never allow the measuring surface to come into "striking" contact with hard object. Follow closely the guidelines below:

- Do not handle or manipulate gage blocks over and above the gage-block set case.

- Spread a cushion sheet over the area where measurements are being taken.

- Do not grab more than one block each time.

Federal Accuracy Grade 2 Tolerance (in micro inches)

Length	+4 or -2
Flatness	±4
Parallelism	±4

Measurement Instruments

Specification:	Digital Micrometer	Mechanical Micrometer	Calipers
Instrumental error	±0.0001 in	±0.0001 in	±0.001 in
Resolution	0.00005 in	0.0001 in	0.0005 in
Quantizing Error	± 1 count	N/A	N/A
Parallax Error	N/A	2 µm	N/A
Parallelism between faces	1µm	1µm	N/A
Flatness of Measuring faces	3µm	1µm	N/A
Measuring Range	0-1 in	0-1 in	0-8 in
Measuring Force	6-10 N, 1.5 to 2 turns of ratchet stop		N/A

B.   Dimensional Measurements

Use the mechanical micrometer and the digital calipers to measure the dimension of an "unknown" object that will be provided to your group. The "unknown" is an aluminum rod (Type 6061-T6) that is manufactured to a certain tolerance (see website for information). Repeat the measurements a minimum of forty times at different locations along the specimen and obtain a representative value (sample mean) of the diameter.

It is assumed that you will be using the Excel program to perform your data reduction. Report just your findings in your log book, and the tape in Excel printouts as needed. Using the measurements and the calibration data perform the tasks below.

LOGBOOK ANALYSIS AND REPORT REQUIREMENTS

Perform the following tasks and report the results:

(a) compute the mean, variance, and standard deviation for the "unknown" diameter;

(b) perform the Chauvenet criterion test (see lecture notes and page 11 of Prof. Chen's notes on the website) and remove a questionable data point; to be statistically correct, after a data

point is removed there is a new "sample" data set, and the mean, variance, and standard deviation must be recomputed before performing another check for questionable data.

(c) after performing the Chauvenet criterion test (no more than twice!) compute again the mean, variance, and standard deviation, remark on any remaining questionable data;

(d) plot a histogram of both data sets (using a method discussed in lecture) after the Chauvenet criterion being performed;

(e) estimate the total error (uncertainty) of your measurements, including both the bias error determined from calibration and the precision error from your dimensional measurements of the unknown based on a 95% confidence level.

(f) Determine if the aluminum rod is within the manufacture's tolerance based on your measurements.

LOG BOOK ASSIGNMENT

A detailed "diary" should be kept in the Log Book for both component (A) and (B). Describe the method of data reduction in detail in your Log Book providing example calculations. Gain experience using the Excel program to do data reduction. A technical report is to be prepared for both (A) and (B).

PRELAB QUESTION

You measure the thickness of a gage block five times using a measurement device having a resolution of 0.001 cm. You measure the following thickness values in cm: 1.100, 1.110, 1.090, 1.095, and 1.105. What is the confidence interval of the true gage block thickness, including all precision and bias errors you can determine from this question statement? Hint: from this example you should know one bias and one precision error.

READING

1. Figliola, R. S. and Beasley, D. E., *Theory and Design for Mechanical Measurements*, Third Ed., Wiley, 2000, pp. 1-28, 109-131 and 149-151.

2. Anonymous, *Mitutoyo Manuals (Nos. 1003, 1038 and 4144)*, Mitutoyo, 1993.

Appendix A: Determining Accuracy and Precision of Measurements: Using Calibration to Estimate Bias and Precision Errors

The write-up below is a guide for determining bias and precision errors in your measurements. In has two parts: Part A deals with calibration of measurements relative to a local standard; here you establish the accuracy of your measurements relative to the standard. Then Part B the total error of measurements are determined, in which the total instrument error established during calibration is combined with the errors determined from the measurements taken.

A) Determining Bias and Precision Errors During Calibration

When calibrating a sensor, you establish the total measurement error which is composed of bias errors and precision errors:

Precision errors are determined based on statistical data, and reflect errors due to random variations in repeated measurements. For example, precision errors can be due to variations in spatial location of a sensor or temporal variations. A low precision error means your repeated measurements fall close together as in the dart board analogy Figure a) shown below taken from Figliola et al., *Theory and Design for Mechanical Measurements*. Note that a small precision error does not necessarily mean your measurements have high accuracy, since they may fall close together but far from the true value.
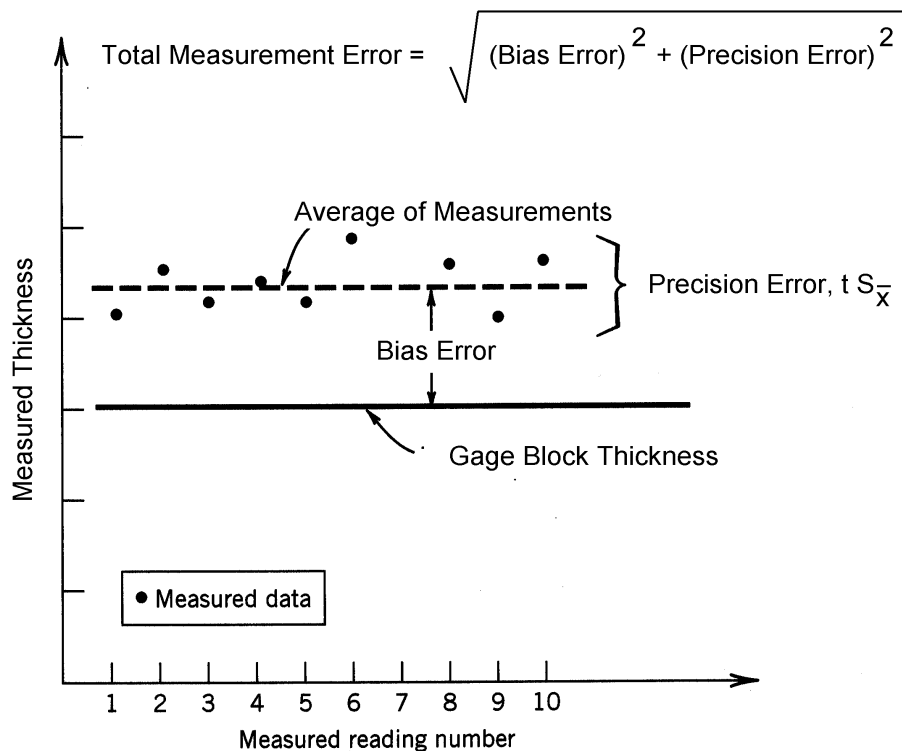

(a) High repeatability gives low precision error but no direct indication of accuracy   (b) High accuracy means low precision and bias errors   (b) Bias and precision errors lead to poor accuracy

Figure   Throws of a dart: illustration of precision and bias errors and accuracy.

Bias errors are errors which have no statistical data associated with them. Resolution error (the so-called zero-order error) is the most obvious bias error; this is stated as +/- 1/2 the resolution of your sensor. Also, during calibration by comparison to a standard, you establish the bias error of your measurement with respect to a standard. This could be established by determining the difference between the mean of your sensor output and the value of the standard you are using for calibration. Another important bias error is the accuracy of your standard. In an absolute sense, the total error of your sensor cannot be less than the error of your standard used in calibrating your sensor. Considering the dart board analogy shown above, Figure a) demonstrates the even thought the precision error is small the bias error maybe large and the accuracy is unknown without a standard for comparison. High accuracy (Figure b) implies that both bias and precision errors are small. In Figure c) bias and precision errors are large and the accuracy is poor.

In the example that follows below, a gage block is used to calibrate the measurements of a micrometer. A gage block is a calibration standard for thickness measurement, it is a block of material, steel in this case, machined to a very high accuracy of thickness. In this case a Federal Grade 2 accuracy gage block is used; it meets certain tolerances for length, flatness and parallelism. The calibrated accuracy of the micrometer can be no greater than the total error of this standard accuracy.

As indicated in the figure below, the total error from the calibration will be the root sum of the squares (RSS) of the bias and precision errors. In the figure, the scatter of ten measured data points lies above the gage block thickness (the true thickness, or true value of the standard). In this figure, the only bias error shown is the average measurement bias errors (it is not the only one considered in the analysis below).

$$\text{Total Measurement Error} = \sqrt{(\text{Bias Error})^2 + (\text{Precision Error})^2}$$

Average of Measurements

Precision Error, $t\, S_{\bar{x}}$

Bias Error

Gage Block Thickness

Measured Thickness

• Measured data

1 2 3 4 5 6 7 8 9 10
Measured reading number

Consider a calibration procedure where forty measurements were made with the micrometer on the gage block.

$N := 40$

The mean thickness measured on the gage block is
$x_{ave} := 0.12621\text{in}$

The standard deviation of the gage block is
$S_x := 0.00014631\text{in}$

The t-statistic for a sample size of 40 is (remember degrees of freedom is N-1)

$t_{39\_95} := 2.02268893$

The standard gage block thickness established the true thickness which is

$x_{true} := 0.12620 in$

From this data we can establish the **bias error of the mean of the measurements** with respect to the standard.

This bias error is:

$\varepsilon_{bias\_mean} := |x_{true} - x_{ave}|$

$\varepsilon_{bias\_mean} = 10 \times 10^{-6} in$

Also, the **precision error of the measurements** is:

$\varepsilon_{precision} := t_{39\_95} \dfrac{S_x}{\sqrt{N}}$

$\varepsilon_{precision} = 4.679 \times 10^{-5} in$

We should always consider the bias error associated with resolution of the measurements. In the case the resolution of the micrometer is

$Resolution := 0.00005 in$

So, the **resolution bias error is**:

$\varepsilon_{resolution} := \dfrac{Resolution}{2}$

$\varepsilon_{resolution} = 2.5 \times 10^{-5} in$

For a calibration analysis such as this, the calibration standard has some errors associated with its accuracy transferred from the "true" dimensional standard. From the standard's documentation, these are from length, flatness and parallelism tolerances for Federal Grade 2 accuracy gage blocks. Taking the largest possible errors in the standard, these errors are conservatively estimated to be:

$\varepsilon_{standard\_length} := 4 \cdot 10^{-6} \cdot in$

$\varepsilon_{standard\_flatness} := 4 \cdot 10^{-6} \cdot in$

$\varepsilon_{standard\_parallelism} := 4 \cdot 10^{-6} \cdot in$

$\varepsilon_{standard\_total} := \sqrt{\varepsilon_{standard\_length}^2 + \varepsilon_{standard\_flatness}^2 + \varepsilon_{standard\_parallelism}^2}$

Therefore the **bias error associated with our local standard**, or the error with respect to the true absolute dimensional measurement is

$\varepsilon_{standard\_total} = 6.928 \times 10^{-6} in$

Note that even if the bias and precision errors of our measurements were identically "zero", we could still not claim our measurements to be more accurate (in an absolute sense) than the error associated with the standard used for the calibration.

Note that the error of the standard is a fraction of our measurement resolution. This is desirable but not always possible. Highly accurate standards usually come at a high price!

In general, if you don't have a standard, your error must be estimated conservatively using information from your measurement system's/sensor's vendor, and any other available data. You should at least include the resolution error.

From this calibration process, the **total error or overall instrument error** of this micrometer measurement is calculated by the RSS of all the errors:

$$\varepsilon_{calibration\_total} = \sqrt{\varepsilon_{bias\_mean}^2 + \varepsilon_{precision}^2 + \varepsilon_{standard\_total}^2 + \varepsilon_{resolution}^2}$$

$$\varepsilon_{calibration\_total} = 5.443 \times 10^{-5} \, in$$

The **accuracy** of these measurements is determined relative to the standard as

$$Accuracy\% := \left(1 - \frac{|\varepsilon_{calibration\_total}|}{x_{true}}\right)$$

$$Accuracy\% = 99.957\%$$

From this point forward, in any error or uncertainty analysis you may use this overall instrument error as a **bias error**. It includes all the errors established during calibration. Note that it includes the resolution error, so you don't need to include that again later. You can include the resolution here or later, but don't account for it twice.

B) Using the Calibrated Error in Measurements

Consider now that you have made forty measurements with the micrometer that you calibrated in Part A). In this case you have measured the diameter of a cylinder.

The average diameter is
$$x_{cyl\_ave} := .32512 \, in$$
$$N := 40$$

The standard deviation is
$$S_x := 0.0003 \, in$$

The t-statistic is
$$t_{39\_95} := 2.02268893$$

You can now, as the experienced experimentalist that you are, state that the true diameter of the cylinder has a confidence interval of

$$\varepsilon_{cylinder} := \sqrt{\varepsilon_{calibration\_total}^2 + \left(t_{39\_95}\frac{S_x}{\sqrt{N}}\right)^2}$$

$$\varepsilon_{cylinder} = 1.103 \times 10^{-4} \, in$$
based on 95% probability.

So the "true" diameter of your cylinder is 0.32512 + or - 0.000057 inches.

Appendix B: Technical Information About Gage Blocks – Errors of Your Standard

The gage blocks are designed to be used where the ambient temperature is 20 ˚C. For accurate measurements, the gage blocks should have reached thermal equilibrium with the ambient temperature before taking measurements. The gage blocks will expand (or contract) at temperatures above (or below) 20 ˚C. This will introduce an error. The gage blocks used in this laboratory (Mitutoyo 516 Series) have a certified Grade 2 accuracy [1].

Handle the gage block carefully and never damage its surface, particularly the measuring face. Do not drop a gage block even onto a soft surface and never allow the measuring surface to come into "striking" contact with hard object. Follow closely the guidelines below:

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